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Chapter 4

Reed-Solomon Codes and the Compact Disc

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1 INTRODUCTION

The compact disc digital audio system, compact disc for short, can be considered as a transmission system that brings sound from the studio into the living room. The sound encoded into data bits and modulated into channel bits is sent along the "transmission channel" consisting of write laser, master disc, user disc, and optical pickup. Imperfections of the disc will produce errors in the recovered data. The nature of the errors leads, in a natural fashion, to the adoption of Reed-Solomon codes. The compact disc system was the first example of the introduction of Reed-Solomon codes in a consumer product. In this chapter, we shall provide a description of the various factors that play a role in the design of the compact disc error control code.

The advantages of digital audio and video recording have been appreciated for a long time and, of course, computers have long been operated in the digital domain. The advent of ever cheaper and faster digital circuitry has made feasible the creation of new devices such as the compact disc and Digital Compact Cassette (DCC) recorder, an impracticable possibility using previous generations of conventional analog hardware. The principal advantage that digital implementation confers over analog systems is that in a well-engineered digital recording system the sole significant degradation takes place at the initial digitization and the fidelity lasts until the point of ultimate failure. In an analog system, sound fidelity is diminished at each stage of signal processing and clearly the number of recording generations is limited. The provision of error-correcting codes, which by necessity work in the digital domain, has made it possible to almost perfectly reconstitute the recorded signal, even in the presence of imperfections in the recording medium and the replay mechanism. It is not an exaggeration to say that, without error-correcting codes, digital audio would not be technically feasible. There are two kinds of errors: those that are distributed randomly among the individual bits, called *random errors*, and those that occur in groups that cover hundreds or even thousands of bits, called *burst errors*. Burst errors, caused by dropouts, are usually the result of surface contamination from fingerprints and scratches on the disc. The above characterization of the recording channel is by necessity qualitative. Any statement beyond truisms such as "some error bursts are longer than others" is speculative and must be handled with great care.

Coding techniques are used in communication systems to improve the reliability and efficiency of the communication channel. The *reliability* is commonly expressed in statistical terms such as the probability of receiving the wrong information, that is, information that differs from what was originally transmitted. Error control is concerned with techniques of delivering information from a source (the sender) to a destination (the receiver) with a minimum of errors. In a digital audio recorder system, the sound signal is digitized in the form of (binary) symbols. The digital data stream so obtained is not directly recorded on tape. In order to make it possible to reliably record the digital data, the data are, prior to recording, translated in two successive steps: (a) error correction code and (b) recording code.[1] The output generated by the recording code is stored on the storage medium in the form of binary physical quantities, for example, pits and lands or positive and negative magnetizations. During readout, the data are obtained via the decoders for the recording code and the error correction code. Schematically the elements of the coding steps in a digital recorder are similar to those of a "point-to-point" communication link. The physical quantities written on the medium are generally very small, and this means that dropouts caused by fingerprints or defects on the medium as well as the methods to cope with them are of the greatest importance. Error correction control is realized by adding extra symbols to the conveyed message. These extra symbols make it possible for the receiver to detect and/or correct some of the errors that may occur in the

[1] In some audio recorders, we may find a third coding step called *source coding*. Source coding is, roughly speaking, a technique to reduce the source symbol rate by removing the redundancy in the source signal. State-of-the-art audio source coders can reduce the symbol rate by a factor of 4 or 5 without sacrificing too much sound fidelity. Source coding is not a part of the compact disc system and therefore is not discussed here.

retrieved message. The main challenge is to achieve the required protection against the inevitable transmission errors without paying too high a price in adding extra symbols (the addition of the extra symbols will lower the effective capacity, say, playing time, of the storage medium). There are many different families of error-correcting codes. Of major importance for recording applications is the family of *Reed-Solomon* (*RS*) *codes*. The reason for their preeminence in recording systems is that they can combat combinations of random as well as burst errors. The success story of RS codes started with its first practical application in digital audio recorders. Pioneering research in this field was conducted by Tanaka et al. in 1978 [13]. Tanaka's design team opted for an RS code constituted by 3-bit symbols, i.e., elements of $GF(2^3)$, and code words of length 8 (symbols). The compact disc, as standardized by Sony and Philips in 1980, was the first example of the widespread introduction of Reed-Solomon codes in the consumer market. Thereafter, Reed-Solomon codes have held undivided sway in digital audio and video storage products such as the DAT recorder [9], the Digital Compact Cassette (DCC) [8], and D1-D2 video recorders [15].

This chapter starts with a brief outline of the system aspects of the compact disc followed by a description of the coding techniques employed. Details of the physical context of optical recording systems are not discussed; the reader is referred to the literature [2].

2 DESCRIPTION OF THE COMPACT DISC SYSTEM

In this section, we shall deal in some detail with the various factors that had to be weighed one against the other in the design of the coding techniques employed in the compact disc system. With its high information density and a playing time of 73 min, the outside diameter of the disc is only 120 mm. Because the disc is so compact, the dimensions of the player can also be small. The way in which the digital information is derived from the analog sound signal gives a frequency characteristic that is flat from 20 to 20,000 Hz. With this digital system the well-known "wow and flutter" of conventional players are a thing of the past.

In the compact disc system, the analog audio signal is sampled at a rate of 44.1 kHz, which is, according to Nyquist's sampling theorem, sufficient for the reproduction of the maximum audio frequency of 20 kHz. The signal is quantized by the method of uniform quantization: the sampled amplitude is divided into equal parts. The number of bits per sample is 32, that is, 16 for the left and 16 for the right audio channel. This corresponds to a signal-to-noise ratio (where the noise is caused by quantization error) of more than 90 dB. The net bit rate is thus $44{,}100 \times 32 = 1.41$ Mbits/s. The audio bits are grouped into blocks of information, called *frames*, each containing six of the original stereo samples. Figure 4-1 represents the complete compact disc system as a "transmission link" that brings the sound from the studio into the living room. The orchestral sound is converted at the recording end into a bit stream B_i, which is recorded on the master disc. The master disc is used as the "pattern" for replicating the discs for the user. The player in the living room derives the bit stream B_0 from the user disc, which in the ideal case should be a facsimile of B_i, and reconverts it to the orchestral sound. The system between COD and DECOD is the actual transmission channel. Figure 4-2 shows the encoding system in more detail. The audio signal is first converted into a stream B_1 of audio bits by means of pulse-code modulation (PCM). The parity bits for error correction and a number of bits for "control and display" are then added to the bit stream [3]. This results in the data bit stream B_2. The modulator converts this stream into channel bits (B_3). The bit stream B_i is obtained by adding a synchronization signal. The bit stream B_i in Figure 4-1 is converted into a signal at P that switches the light beam from the write laser on and off. The channel should be of high enough quality to allow the bit stream B_i to be reconstituted from the read signal at Q. To achieve this quality, all the stages in the transmission path must meet exacting requirements, from the recording on the master disc, through the disc manufacture, to the actual playing of the disc. The quality of the channel is determined by the player and the disc: these are mass-produced, and the tolerances cannot be made unacceptably small. A general picture can be obtained from Table 1, which gives the manufacturing tolerances of a number of relevant parameters, both for the player and for the disc. With properly manufactured players and discs, the channel quality can still be impaired by dirt and scratches forming on the discs during use. By its nature, the optical system is fairly insensitive to these damages [3,6], and any errors they may introduce can nearly always be corrected or masked. In the following, we shall see that the recording code also helps to reduce the sensitivity of imperfections.

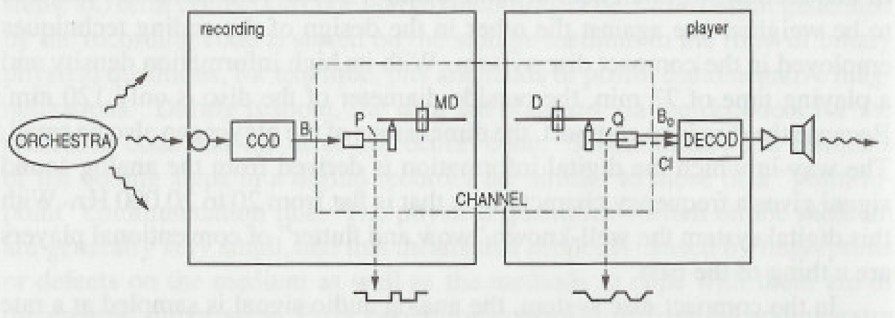

Figure 4-1. Compact disc system considered as a transmission system that brings sound from the studio into the living room. The transmission channel between the encoding system (COD) at the recording side and the decoding system (DECOD) in the player "transmits" the bit stream B_i to DECOD via the write laser, the master disc (MD), the disc manufacture, the disc (D) in the player, and the optical pickup; in the ideal case B_0 is the same as B_i. The bits of B_0, as well as the clock signal (Cl) for further digital operations, have to be detected from the output signal of the pickup unit at Q. Taken from [6].

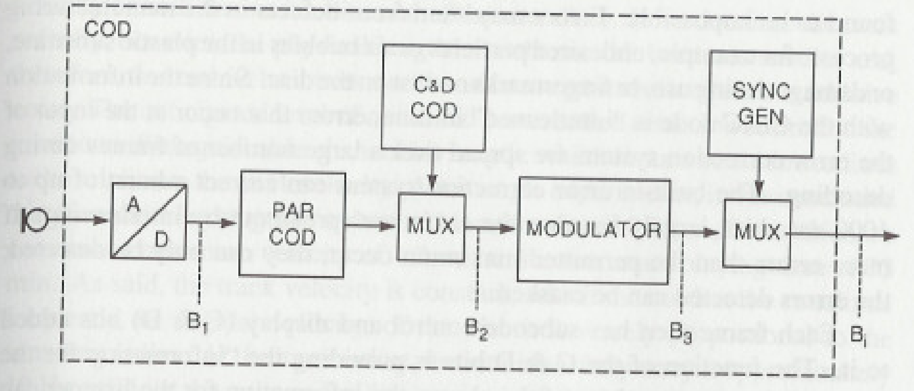

Figure 4-2. Encoding system (COD in Figure 4-1). The system is highly simplified here. In practice there are two channels for stereo which together supply the bit stream B_1. The bit steam B_1 is supplemented by parity and control display bits (B_2) translated by the encoder (B_3) and provided with synchronization signals (B_i). MUX: multiplexers. Taken from [6].

TABLE 1 Manufacturing Tolerances

Player	Objective-lens tilt ±0.2°
	Tracking ±0.1 μm
	Focusing ±0.5 μm
Disc	Thickness 1.2 ±0.1 mm
	Flatness ±0.6°
	Pit-edge positioning ±50 nm
	Pit depth 120 ±10 nm

Successive blocks of audio bits have blocks of parity symbols added to them in accordance with a coding technique called cross-interleaved Reed-Solomon code (CIRC) [11,14] which will be detailed in Section 4. The ratio of the number of bits before and after this operation, the *rate* of the CIRC code, is 3:4. The parity symbols can be used here to correct errors. Due to the nature of optical recording, it is impossible to fully correct all possible error patterns. A well-engineered system must be able to offer a graceful degradation during errors. Error concealment is required if error correction is found to be impossible. Errors may stem from defects in the manufacturing process, for example, undesired particles or air bubbles in the plastic substrate, or damage during use, or fingermarks or dust on the disc. Since the information with the CIRC code is "interleaved" in time, errors that occur at the input of the error correction system are spread over a large number of frames during decoding. The built-in error correction system can correct a burst of up to 4000 data bits, largely because the errors are spread out by interleaving. If more errors than the permitted maximum occur, they can only be detected; the errors detected can be masked.

Each frame then has subcode control and display (C & D) bits added to it. The function of the C & D bits is providing the "information for the listener." In some versions of the player, the information for the listener can be represented on a display and the different sections of the music can be played in the order selected by the user. After the previous operation the bits are called *data bits*. Next the bit stream is encoded into a sequence of binary symbols, called *channel bits*, which are suitable for storage on the disc. The eight-to-fourteen modulation (EFM) code is used for this. Under EFM coding rules, data blocks of 8 bits are translated into blocks of 14 channel bits that have special properties. The blocks of 14 bits are linked by three *merging bits*. The ratio of the number of data bits entering the EFM encoding stage and the number of channel bits leaving it is 8:17.

For synchronization of the bit stream an identical synchronization pattern consisting of 27 channel bits is added to each frame. The total bit rate after all these manipulations is approximately 4.32×10^6 channel bits/s. Table 2 gives a survey of the successive operations with the associated bit rates and their names.

TABLE 2 Successive Signals, Associated Bit Rates, and Operations During Processing of an Audio Signal

Name	Bit rate (Mbits/s)	Operations
Audio signal		PCM (44.1 kHz)
Audio bit stream	1.41	CIRC
Data bit stream	1.94	EFM (+ sync. bits + C & D)
Channel bit stream	4.32	

As the track velocity is constant, the playing time of a disc is equal to the track length divided by the track velocity. The track length is simply the useful disc area divided by the track pitch. For the given disc diameter, 120 mm, and track pitch, 1.6 μm, it follows that the track length is about 5.3 km. The playing time increases if we decrease the track velocity in the system (the track

velocity of both the master disc and of the user disc). If we decrease the track velocity, however, the physical length of a channel bit becomes smaller and, therefore, the readout becomes more sensitive to perturbations such as additive noise. The anticipated level of noise and perturbation sets a lower limit to the minimum distinguishable channel bit length, which in turn sets a lower limit to the minimum the track velocity. After ample experiments, it was concluded that the minimum length of a channel bit on the disc is 0.3 μm, from which we conclude a minimum velocity of 1.2 m/s and a maximum playing time of 73 min. As said, the track velocity is constant, which means that the rotational frequency, or angular velocity, of the disc is inversely proportional to the scanning radius. At the inner radius, the rotational frequency is approximately 10 Hz, while at the outer radius it is approximately 3 Hz. During readout, the rotational frequency is automatically adjusted by the player's electronics.

3 EFM RECORDING CODE

The EFM code is a member of the family of *run-length-limited codes*. In this context, the number of consecutive zeros or ones in a (binary) sequence is known as *run length*. A run-length-limited sequence is a sequence of binary symbols characterized by two parameters, $d + 1$ and $k + 1$, which stipulate the minimum and maximum run length, respectively, that may occur in the sequence. The parameter d controls the highest transition frequency and thus has a bearing on intersymbol interference when the sequence is transmitted over a bandwidth-limited channel. In the transmission of binary data it is generally desirable that the received signal is self-synchronizing or self-clocking. Timing is commonly recovered with a phase-locked loop that adjusts the phase of the detection instant according to observed transitions of the received waveform. The maximum run-length parameter k ensures adequate frequency of transitions for synchronization of the read clock. The grounds on which d and k values are chosen, in turn, depend on various factors such as the channel response, the desired data rate (or information density), and the jitter and noise characteristics. The design considerations underlying a certain choice are outside the scope of this chapter. The interested reader is referred to [7], which also provides more details of the EFM code and other run-length-limited codes.

Figure 4-3 gives a schematic general picture of the bit streams in the encoding system. The information is divided into frames. One frame contains 6 sampling periods, each of 32 audio bits (16 bits for each of the two audio channels). These 32 audio bits are divided into 4 words of 8 bits. The bit stream B_1 thus contains 24 words per frame. In B_2, 8 parity words and 1 C & D word have been added to each frame, resulting in 33 data words. The modulator translates each byte (8 bits) into a new word of 14 bits. Added to these are 3 merging bits, for reasons that will appear shortly. After the addition of a synchronization pattern of 27 bits to the frame, the bit stream B_i is obtained. B_i therefore contains $33 \times 17 + 27 = 588$ channel bits per frame. Finally, B_i is converted into a control signal for the write laser. It should be noted that in B_i a 1 or a 0 does not mean "pit" or "land," but a 1 indicates a pit edge. The information is thus completely recorded by the position of the pit edges; it therefore makes no difference to the decoding system if pit and land are interchanged on the disc.

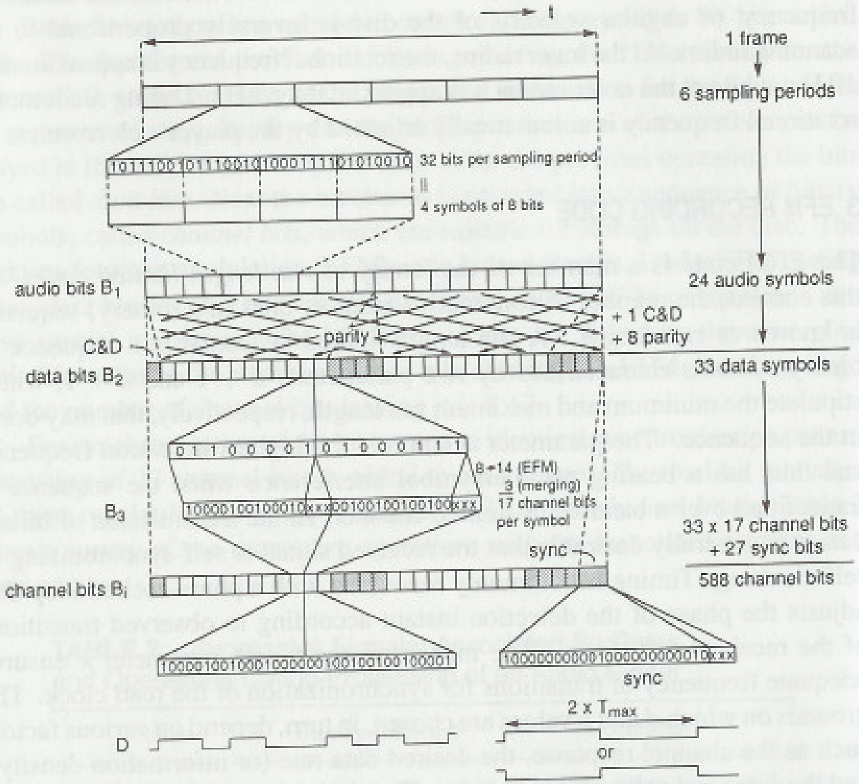

Figure 4-3. Bit streams in the encoding process (Figure 4-2). The information is divided into frames; the figure shows one frame of the successive bit streams. Six audio sampling periods constitute one frame, each sampling period giving 32 bits (16 for each of the two audio channels). The EFM code translates 8 data bits B_2 into 14 channel bits, to which 3 merging bits are added. At B_3 the frames are marked with a synchronization pattern of the form illustrated (bottom right); the final result is the channel bit stream B_i used for writing on the master disc in such a way that each 1 indicates a pit edge (D). Taken from [6].

Opting for the translation of series of 8 bits following the division into words in the parity coding has the effect of avoiding error propagation. One channel bit error will spoil an entire word, but because of the correspondence between channel words and data words, never more than one word. If a different recording code is used, in which the data bits are not translated in groups of 8 but in groups of 6 or 10, say, then the bit stream B_2 is in fact first divided up into 6- or 10-bit channel words. Although one channel-bit error then spoils only one channel word, it usually spoils two of the original 8-bit words. A quantitative description of the above error propagation phenomenon has been given by Blaum [1]. Under EFM rules, the data bits are translated 8 at a time into 14 channel bits, with a minimum run-length parameter d of 2 and a maximum run-length parameter k of 10 channel bits (this means at least 2 and at the most 10 successive zeros in B_i). The grounds for chosing these specific values of d and k are not easily explained. This choice came about more or less as follows. The minimum run-length parameter $d = 2$, with about 16 channel bits on 8 data bits, is about the optimum for the compact disc system. A simple calculation shows that at least 14 channel bits are necessary for the reproduction of all the 256 possible words of 8 data bits under the conditions $d = 2$, $k = 10$. The choice of the maximum run-length parameter k was dictated by the fact that a larger choice does not make things very much easier, whereas a smaller choice does create far more difficulties. With 14 channel bits it is possible to make up 267 words that satisfy the minimum run-length condition $d = 2$. Since we require only 256, we omitted 10 to make it possible to limit the maximum run length to $k = 10$. One other code word was deleted more or less at random. In order to reduce the complexity of the decoder logic, the relationship between (output) data patterns and (input) code patterns has to be optimized. The codebook was compiled with the aid of computer optimization in such a way that the translation in the player can be carried out with the simplest possible circuit, i.e., a circuit that contains the minimum of logic gates. In the compact disc player, the EFM conversion can be performed with a logic array of approximately 50 logic functions. Part of the EFM coding table is presented in Table 3, which shows the decimal representation of the 8-bit source word (left column) and its 14-bit channel representation (right column). The merging bits are primarily intended to ensure that the run-length conditions continue to be satisfied when the code words are merged. If the run length is in danger of becoming too short, we choose zeros for the merging bits; if it is too long, we choose a one for one of them.

TABLE 3 Part of the EFM Coding Table

Data	Code	Data	Code
100	01000100100010	114	10010010000010
101	00000000100010	115	00100000100010
102	01000000100010	116	01000010000010
103	00100100100010	117	00000010000010
104	01001001000010	118	00010001000010
105	10000001000010	119	00100001000010
106	10010001000010	120	01001000000010
107	10001001000010	121	00001001001000
108	01000001000010	122	10010000000010
109	00000001000010	123	10001000000010
110	00010001000010	124	01000000000010
111	00100001000010	125	00001000000010
112	10000000100010	126	00010000100010
113	10000010000010	127	00100000000010

4 THE CIRC CODE

CIRC code (cross-interleaved Reed-Solomon code) is the name of the error control code used in the CD system. The system requirements are

- High random error correctability
- Long burst error correctability
- In case burst correction capability is exceeded, we still have good concealment possibility
- Simple decoder strategy possibility with reasonable sized external random access memory
- Low redundancy
- Possibility for future introduction of four audio channels without major changes in the format.

The errors found in the CD system are a combination of a random and bursty character, and in order to alleviate the load on the error control code some form of interleaving is required. The interleaving scheme is tailored to the specific requirements of the compact disc system. In particular, the adopted cross-interleaving technique will make it possible to effectively mask errors if correction is found to be impossible. Depending on the magnitude of the error to be concealed, this can be done by interpolating or by muting the audio signal. If a large error has occurred and a single audio sample cannot be reconstituted by the error control circuitry, it is possible to obtain an approximation to it by interpolating the neighboring audio samples. The concealment will make errors almost inaudible, and as a result, it offers a graceful degradation of the sound quality. Specifically, most of the sharp, temporary degradations of the audio signal, "clicks," are avoided. The judicious positioning of the left and right stereo channels as well as the audio samples on even- or odd-number instants within the interleaving scheme are key parameters to the success of the concealment strategy. There are a number of interleaved structures used in the CD, each of which makes it possible to correct and detect errors with a minimum of redundancy. The CIRC interleaving scheme will be discussed in the next section.

5 THE ART OF INTERLEAVING

There are a number of different ways in which interleaving can be performed. The simplest interleaving method is termed *block interleaving*. In block interleaving, data are written into a memory that is organized as an $n_1 \times n_2$ matrix. The data are written column-wise and read row-wise. The code words have a length of n_2 symbols. It can easily be verified that if the code is single-error-correcting, bursts of a length at most n_1 symbols can be corrected. Note that the required memory capacity of the interleaver is $n_1 \times n_2$ symbols. The compact disc uses a more effective type of interleaver called a *cross-interleave*. This type of interleave structure, due to Ramsey [12], is also known as a periodic or convolutional interleaver [5]. The essence of the convolutional interleave can be understood from Figure 4-4. Here, code words are constituted by four symbols $\{w_i \dots w_{i+3}\}$. Before transmission, the symbols of the code words are multiplexed over four delay lines with differing delays. The outputs of the delay lines are combined (demultiplexed) and forwarded to the channel. At the receiver end, the data are subjected to the inverse operation. The CIRC code employs a similar approach of interleaving. A schematic description of the CIRC encoder is displayed in Figure 4-5. The error control code used in the CD system employs not one but two Reed-Solomon codes (C_1, C_2), which are interleaved cross-wise. In particular, the synergy of the two RS codes gives excellent results. For code C_1 we have $n_1 = 32$, $k_1 = 28$, and for C_2 we have $n_2 = 28$, $k_2 = 24$. The code symbols are 8 bits long, i.e., elements of GF(2^8). Clearly, the rate of the CIRC code is $(k_1/n_1)(k_2/n_2) = 3/4$. For both of the codes the minimum distance is 5, which makes it possible to directly correct a maximum of two errors in one code or to make a maximum correction of four erasures. The various steps of interleaving can be observed in Figure 4-5. Each frame of information accommodates 12 right and left channel audio samples, denoted by R and L. Each 16-bit sample is divided into 2 bytes indicated by W. The even- and odd-number audio samples are separated by subjecting them to a delay of two

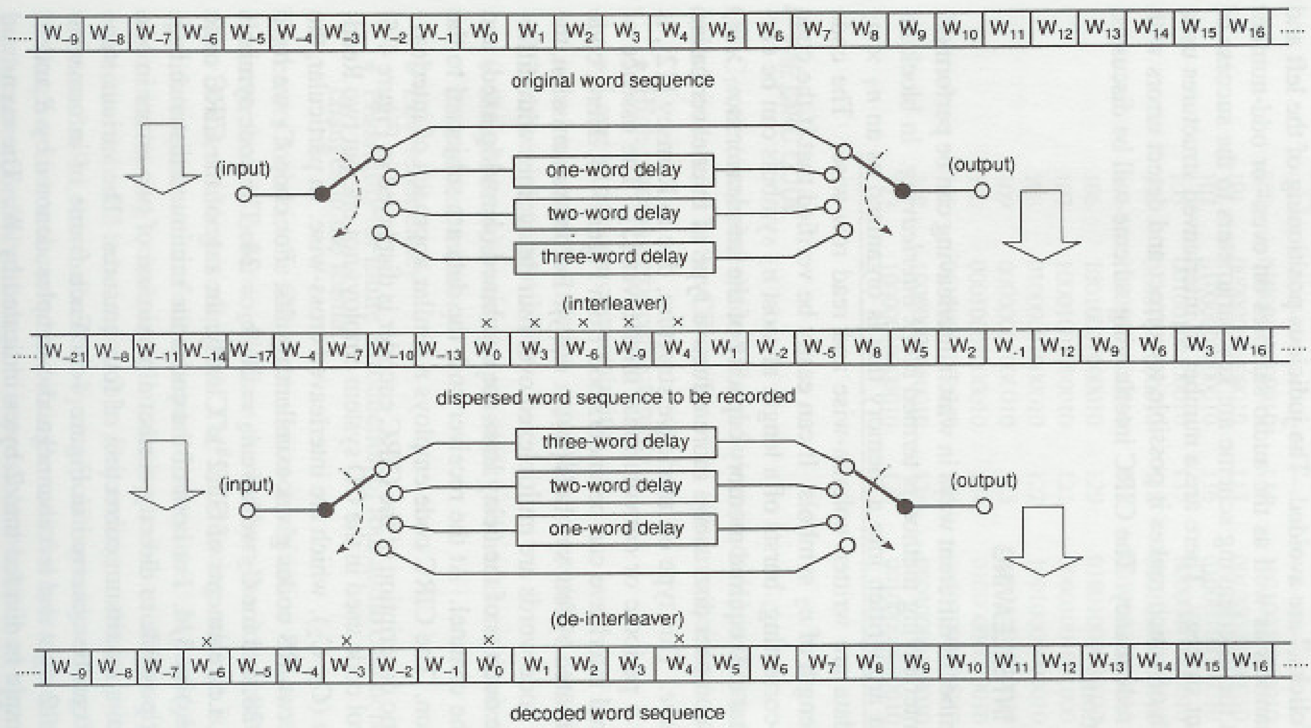

Figure 4-4. Simple interleave and deinterleave scheme using delay lines of differing lengths. As an example, we assume that a burst will damage w_0, w_{-3}, w_{-6}, and w_{-9}. Errors made during transmission are denoted by *. The example shows that correction of four consecutive errors is possible even with a one-error correcting code.

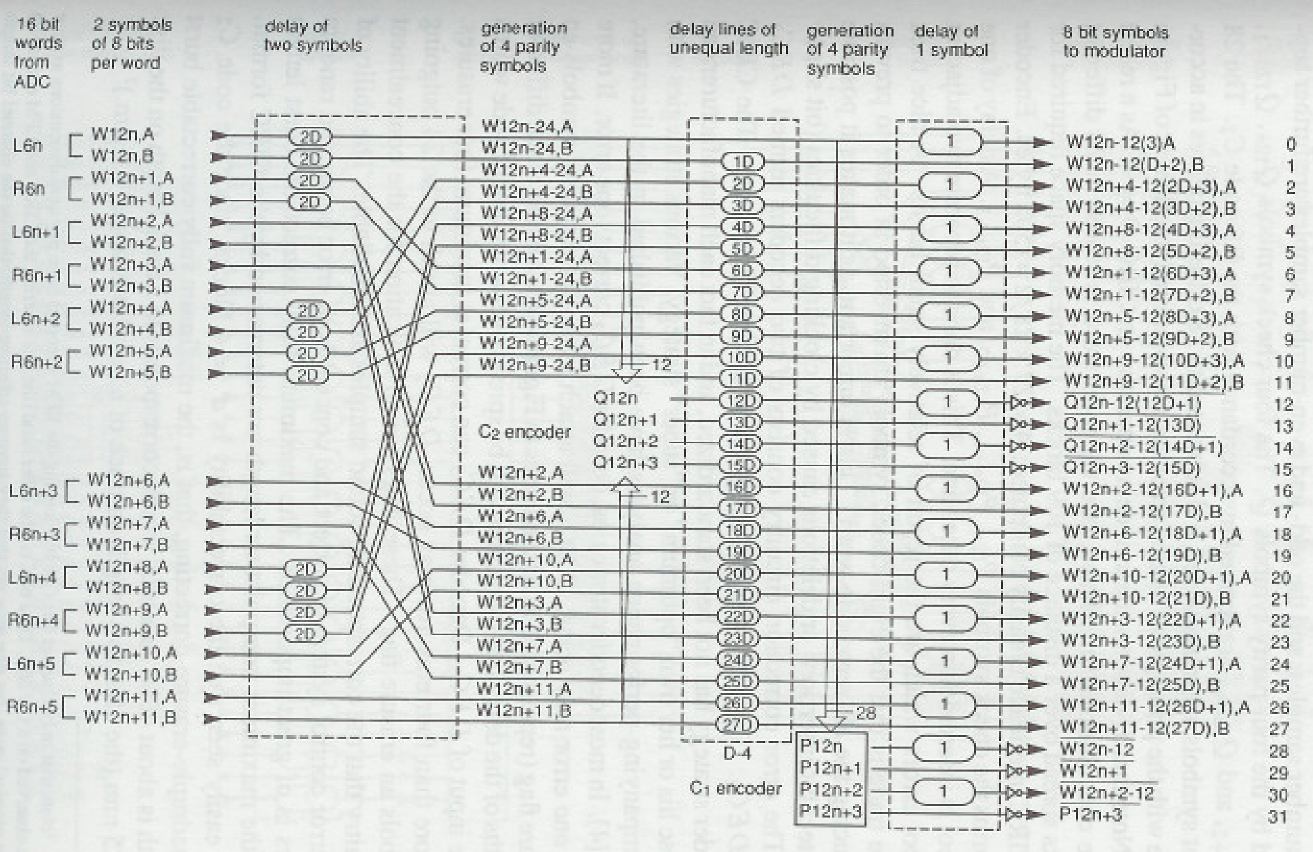

Figure 4-5. Block diagram of CIRC encoder.

symbols. This makes it much easier to conceal uncorrectable errors. The 24 symbols in a frame are regrouped so as to give a separation of the even- and odd-number samples, and the odd- and even-number samples are further separated by the four parity symbols Q. The four check symbols Q_{12n}, Q_{12n+1}, Q_{12n+2}, and Q_{12n+3} are formed in accordance with the RS code C_2. The 28 output symbols are multiplexed and subjected to 28 differing delays in accordance with the principle of operation of the convolutional interleaver of Figure 4-4. Note that D denotes the unit delay operator. In CIRC $D = 4$. As a result of the convolutional interleave, one C_2 code word is stored in 28 different blocks spread over a distance of 109 blocks. The memory space required for the CIRC interleave is easily computed: $4 \times 27 \times 28/2 = 1502$ bytes. Encoder C_2 forms the check symbols P_{12n}, P_{12n+1}, P_{12n+2}, and P_{12n+3}. A delay of one symbol is inserted into every other line. This is done to separate two adjacent symbol errors which are the result of relatively small bursts. A nice trick of the trade is that the eight check symbols are inverted in order to prevent producing the all-zero code word. This is important, as it makes it possible to detect bit insertions or deletions caused by clocking difficulties (bit slip).

The error correction circuitry consists of two decoders termed DEC_1 and DEC_2. They are schematically represented in Figure 4-6. The CIRC decoder strategy has not been standardized, and in fact each manufacturer can choose his or her own "optimum" or "super" strategy. Various strategies with accompanying performance analyses have been published in the literature, e.g., [4]. In most decoder strategies, decoder DEC_1 corrects one error. If more than one error may occur, DEC_1 will attach to the 28 outgoing symbols an erasure flag (represented as dashed lines in Figure 4-6). Owing to the different lengths of the delay lines, erasures will be spread over a number of code words at the input of DEC_2. Decoder DEC_2 can correct at maximum four erasures. If more than four erasures may occur, DEC_2 will attach to the 24 outgoing symbols an erasure flag. These flags are an indication for the concealment circuitry that the corresponding sound samples are unreliable. The ability of the error control system to correct and conceal both error bursts and random errors is of great importance. The maximum fully correctable burst length and the maximum interpolation length are determined by the CIRC format. It is easily seen that the correctability is $d = 4$ blocks, since the code C_2 is quadruple-erasure-correcting, that is, the maximum fully correctable burst length is about 4000 data bits, which corresponds to a track length on the disc of 2.5 mm (the effective length on track of a data bit is about 0.6 μm.)[2]

[2] Note that only channel bits (the bits leaving the EFM encoder) have physical dimensions as they can be observed on a CD disc. Data bits, on the other hand (the bits entering the CIRC and EFM encoders) are not physical units, but one may say that they have an effective length, that is, the channel bit length divided by the code rate.

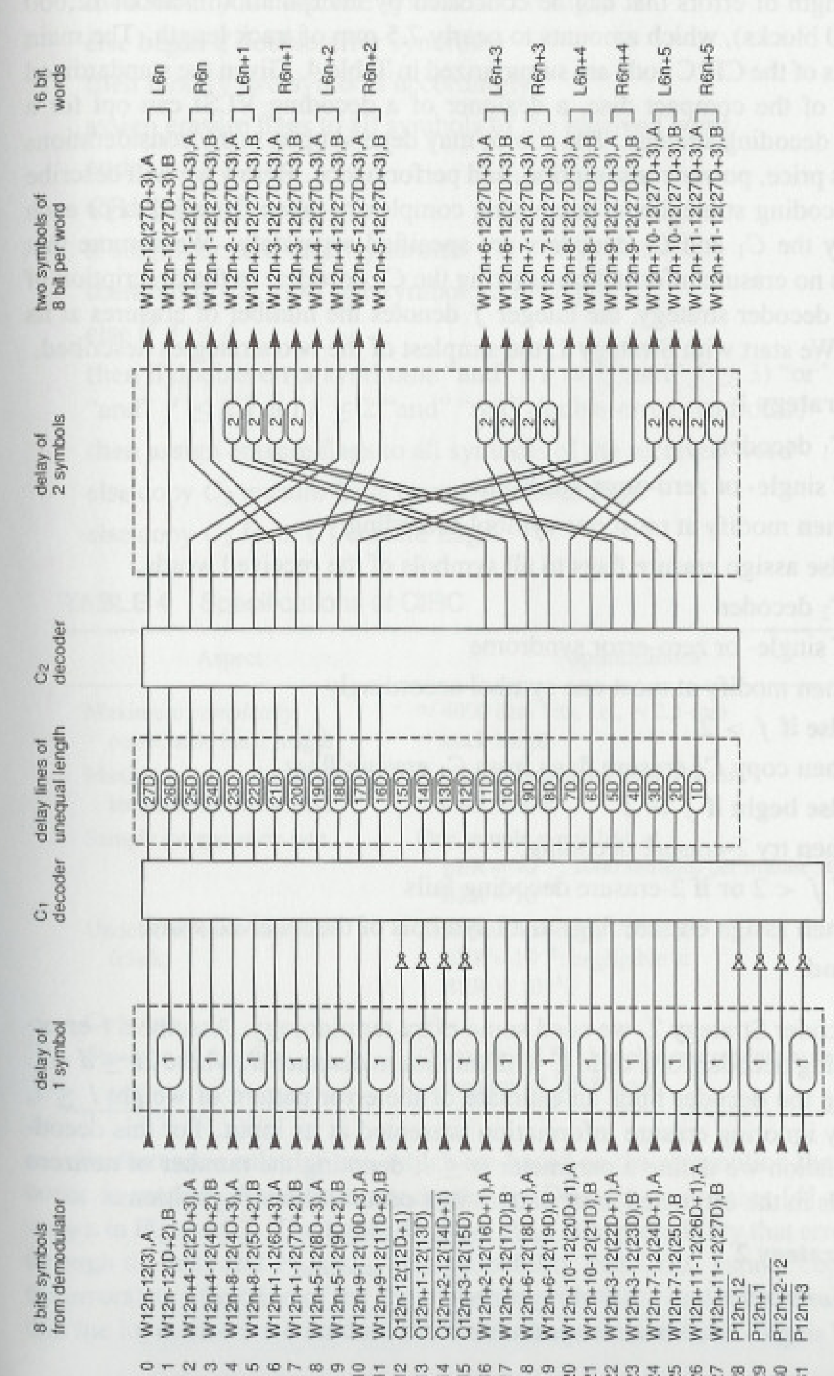

Figure 4-6. Block diagram of CIRC decoder.

The length of errors that can be concealed by interpolation is about 12,000 bits (50 blocks), which amounts to nearly 7.5 mm of track length. The main features of the CIRC code are summarized in Table 4. Given the standardized format of the compact disc, a designer of a decoding VLSI can opt for a certain decoding strategy. The choice may depend upon many considerations such as price, power consumption, and performance. Below we will describe two decoding strategies of increasing complexity taken from [4]. For each strategy the C_1 and C_2 decoders are specified separately. We assume that there is no erasure information entering the C_1 decoder. In the description of the C_2 decoder strategy, the integer f denotes the number of erasures at its input. We start with Strategy 1, the simplest of the two strategies described.

Strategy 1

C_1 decoder
if single- or zero-error syndrome
then modify at most one symbol accordingly
else assign erasure flags to all symbols of the received words.
C_2 decoder
if single- or zero-error syndrome
then modify at most one symbol accordingly
else if $f > 2$
then copy C_2 erasure flags from C_1 erasure flags
else begin if $f = 2$
then try 2-erasure decoding;
if $f < 2$ **or if** 2-erasure decoding fails
then assign erasure flags to all symbols of the received word;
end.

For decoder Strategy 2, we need some extra terminology. Assume a t-error-correcting decoder for a code C with minimum distance d, where $2t \leq d - 1$. Assume the decoder finds an estimate of the error pattern of weight $l \leq t$, initially ignoring erasure information presented at its input. For this decoding situation we define a parameter $v \leq l$, denoting the number of nonzero symbols in the estimated error pattern that occur in erasure positions.

Strategy 2

C_1 decoder
if single- or zero-error syndrome

then modify at most one symbol accordingly
else begin if double-error syndrome
then modify two symbols accordingly;
assign erasure flags to all symbols of the received word.
end
C_2 decoder
if single- or zero-error syndrome
then modify at most one symbol accordingly
else if $f \leq 4$
then if double-error syndrome "and" [($v = 1$ "and" $f \leq 3$) "or" ($v = 0$ "and" $f \leq 2$)] or ($f \leq 2$ "and" "not" double-error syndrome)
then assign erasure flags to all symbols of the received word
else copy C_2 erasure flags from C_1 erasure flags
else copy C_2 from C_1 erasure flags.

TABLE 4 Specifications of CIRC

Aspect	Specifications
Maximum *completely* correctable burst length	$\approx$ 4000 data bits, i.e., $\approx$ 2.5-mm track length
Maximum interpolatable burst length in the *worst* case	$\approx$ 12,300 data bits, i.e., $\approx$ 7.7-mm track length
Sample interpolation rate	One sample every 10 h at BER = 10^{-4}; 1000 samples per minute at BER = 10^{-3}
Undetected error samples (click)	Less than one every 750 h at BER = 10^{-3}; negligible at BER $\leq 10^{-4}$
Code rate	3/4
Structure of decoder	One special LSI chip plus one random-access memory of 2048 bytes

Performance calculations, which are based on the assumption that errors occur at random, have been conducted by Driessen and Vries [4] and are shown in Figure 4-7. The figure displays P_{click} (the probability that errors slip through the net) and $P_{\text{interpolate}}$ (the probability that CIRC cannot cope with the errors) as a function of the channel error probability under the assumption that the input errors are random (no burst errors are assumed).

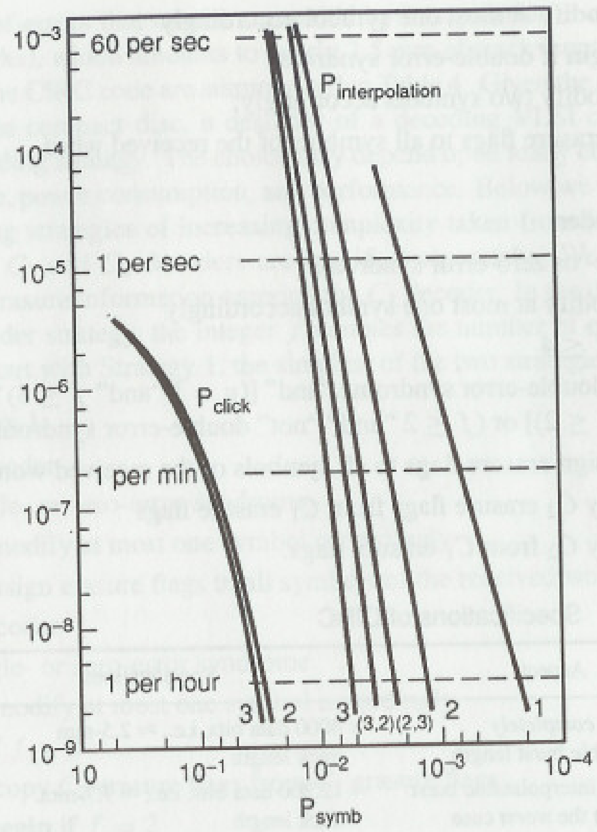

Figure 4-7. Performance curves for several decoding strategies. The quantity P_{click} is the probability that errors pass without being detected, and $P_{interpolate}$ is the probability that errors detected but cannot be corrected as they are beyond the capacity of the CIRC error control system.

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